ters for maximum output power. The oscillator uses a 2×10 μm^2 emitter AlGaAs/GaAs HBT fabricated by means of a pattern inversion technology. The HBT has a base current $1/f$ noise power density lower than 1×10^{-20} A^2/Hz at 1 kHz, and lower than 1×10^{-22} A^2/Hz at 100 kHz, for a collector current of 1 mA. The monolithic oscillator which operates over 20–28 GHz band has a phase noise of -80 dBc/Hz at 100 kHz off carrier when operated at 26.6 GHz. These results indicate the applicability of the HBT's to low-phase-noise monolithic oscillators at microwave and millimeter wave frequencies, where both Si bipolar transistors and GaAs FET's are absent.

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Noise Power Sensitivities and Noise Figure Minimization of Two-Ports with Any Internal Topology

Janusz A. Dobrowolski

Abstract —**A theoretical foundation is presented for the efficient CAD-oriented computation of first-order noise power sensitivities of networks with respect to network parameters. Application to the CAD of low-noise circuits with minimum noise figure using efficient gradient optimization methods is envisaged. The approach is applicable to circuits with any internal topology composed of any number of passive linear multiports and active linear two-ports. It is based on the scattering matrix description for circuit elements and wave representation for noise.**

I. Introduction

A recent paper by the author [1] presented a CAD-oriented noise analysis method for linear two-ports with absolutely general internal topology. The method allows noise figure computation of circuits which are composed of any number of passive linear multiports and active linear two-port devices.

The purpose of this paper is to present a computer-aided method for noise power sensitivity analysis of circuits of any topology. The noise power sensitivities are applicable to noise figure minimization of two-ports with any internal topology using gradient optimization methods. A scattering matrix and a wave representation are used for circuit and noise descriptions [2]–[4]. The approach may be used for noise performance optimization of such circuits as distributed amplifiers and amplifiers with any topology of feedback.

II. Noise Analysis

For a kth noisy element of a general circuit we can write the equation [1], [2]

$$B^{(k)} = S^{(k)}A^{(k)} + B_N^{(k)} \tag{1}$$

where $S^{(k)}$ is the scattering matrix of the element, $A^{(k)}$ and $B^{(k)}$ are the vectors of ingoing and outgoing noise waves at its ports, and $B_N^{(k)}$ is the vector of mutually correlated noise wave sources which represent noise generated in the element.

For a circuit composed of m elements (multiports) connected together by their ports (Fig. 1), we can write the following set of equations [1]:

$$WA = B_N \tag{2}$$

where

$$A = \begin{bmatrix} A^{(1)} \\ A^{(2)} \\ \vdots \\ A^{(m)} \end{bmatrix} \qquad B_N = \begin{bmatrix} B_N^{(1)} \\ B_N^{(2)} \\ \vdots \\ B_N^{(m)} \end{bmatrix} \tag{3}$$

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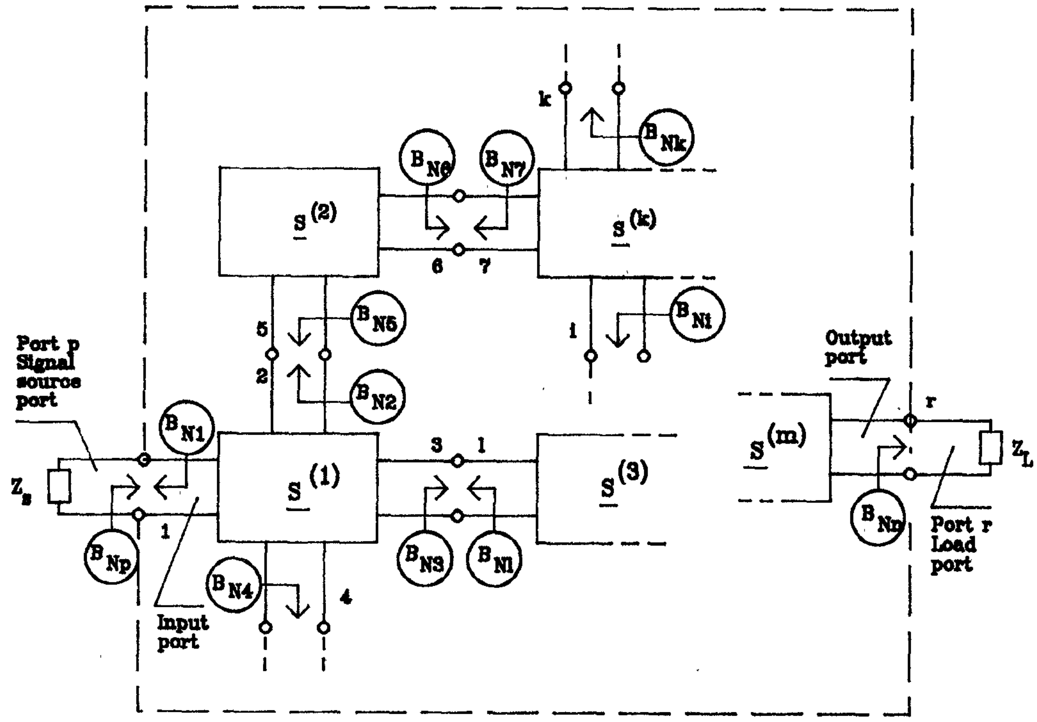

Fig. 1. Equivalent circuit of a noisy network with noiseless elements and noise wave sources at each port.

and

$$W = \Gamma - S \tag{4}$$

is the connection scattering matrix of the circuit [6].

In (3) $A^{(k)}$, $k = 1, 2, \cdots, m$, are vectors of ingoing noise waves at ports of the kth element, and $B_N^{(k)}$, $k = 1, 2, \cdots, m$, are the vectors of mutually correlated noise wave sources, one source at each port, representing noise generated in the element.

Because

$$A = W^{-1} B_N \tag{5}$$

a correlation matrix of the ingoing noise waves at all circuit ports is

$$\overline{AA^\dagger} = W^{-1}\overline{B_N B_N^\dagger}(W^{-1})^\dagger = W^{-1}C(W^{-1})^\dagger \tag{6}$$

where the bars indicate the statistical averages and the daggers the complex conjugate transpose of the vectors and matrices. In (6)

$$C = \overline{B_N B_N^\dagger} = \begin{bmatrix} C^{(1)} & 0 & \cdots & \cdots & 0 \\ 0 & C^{(2)} & & & \cdot \\ \vdots & & \ddots & & \vdots \\ \vdots & & & C^{(k)} & 0 \\ 0 & \cdot & \cdots & 0 & C^{(m)} \end{bmatrix} \tag{7}$$

is the correlation matrix of the noise wave sources representing noise in all circuit elements. $C^{(1)}, C^{(2)}, \cdots, C^{(m)}$ are correlation matrices of the noise wave sources of individual circuit elements. The correlation matrices C of active two-ports such as FET's may be calculated using a set of noise parameters such as F_{em}, $\Gamma_0 = \mathrm{Re}\,\Gamma_0 + j\,\mathrm{Im}\,\Gamma_0$, and N obtained through measurements [7]–[10].

Noise wave correlation matrices of lossy passive multiports (which generate only thermal noise) are defined by the relation [11]

$$C = kT\,df(I - SS^\dagger) \tag{8}$$

where k is Boltzmann's constant, T is the physical temperature of the multiport, df is the frequency bandwidth, I is an identity matrix, and S is the scattering matrix of the multiport.

If r is the number of a load impedance port of the analyzed circuit, then the noise power dissipated in the load is

$$P_N = \left(\overline{AA^\dagger}\right)_{rr}\left(1 - |S_{rr}|^2\right) = N_r\left(1 - |S_{rr}|^2\right) \tag{9}$$

where $N_r = (\overline{AA^\dagger})_{rr}$ is the rth diagonal element of the correlation matrix $\overline{AA^\dagger}$, and

$$S_{rr} = \frac{Z_L - Z_r^*}{Z_L + Z_r} \tag{10}$$

is the reflection coefficient at the output port load Z_L with respect to the reference impedance Z_r of the load impedance port.

The evaluation of N_r can be derived easily from (6). In fact, if only the rth diagonal element of the correlation matrix $\overline{AA^\dagger}$ has to be determined, than by letting $\mathcal{B}_r$ be a vector whose elements are all zeros except 1 in the position r,

$$\mathcal{B}_r^T = [0, \cdots, 0, 1, 0, \cdots, 0] \tag{11}$$

we have

$$\begin{aligned} N_r = \left(\overline{AA^\dagger}\right)_{rr} &= \mathcal{B}_r^T \overline{AA^\dagger} \mathcal{B}_r \\ &= \mathcal{B}_r^\dagger W^{-1} C (W^\dagger)^{-1} \mathcal{B}_r \\ &= \left[(W^\dagger)^{-1}\mathcal{B}_r\right]^\dagger C (W^\dagger)^{-1} \mathcal{B}_r. \end{aligned} \tag{12}$$

The relation (12) may be also written in the form

$$N_r = (\overline{AA^\dagger})_{rr} = \mathcal{A}^\dagger C \mathcal{A} \tag{13}$$

where a vector

$$\mathcal{A} = (W^\dagger)^{-1}\mathcal{B}_r = (W^{T*})^{-1}\mathcal{B}_r \tag{14}$$

is the solution vector of a system of equations whose coefficient matrix is equal to the complex transpose of the connection scattering matrix W of the analyzed circuit and having $\mathscr{B}_r$ as its right-hand-side vector.

III. Noise Power First-Order Sensitivities

Suppose that a parameter p in a given noisy network is to be varied without affecting its topology. Parameter variation will affect the noise power delivered to the load at the rth port of the circuit. Differentiating (12) with respect to p and using the relation $\partial W/\partial p = -\partial S/\partial p$, we have

$$\begin{aligned}\frac{\partial N_r}{\partial p} &= \frac{\partial\left(\overline{AA^\dagger}\right)_{rr}}{\partial p} \\ &= \mathscr{B}_r^\dagger W^{-1}\left[\frac{\partial S}{\partial p}W^{-1}C + \left(\frac{\partial S}{\partial p}W^{-1}C^\dagger\right)^\dagger\right](W^\dagger)^{-1}\mathscr{B}_r \\ &\quad + \mathscr{B}_r^\dagger W^{-1}\frac{\partial C}{\partial p}(W^\dagger)^{-1}\mathscr{B}_r \\ &= \mathscr{A}^\dagger\left[\frac{\partial S}{\partial p}W^{-1}C + \left(\frac{\partial S}{\partial p}W^{-1}C^\dagger\right)^\dagger\right]\mathscr{A} \\ &\quad + \mathscr{A}^\dagger\frac{\partial C}{\partial p}\mathscr{A}. \end{aligned} \tag{15}$$

Because noise correlation matrices are Hermitian matrices ($C = C^\dagger$) [12], (15) simplifies to

$$\frac{\partial N_r}{\partial p} = 2\,\mathrm{Re}\left\{\mathscr{A}^\dagger\frac{\partial S}{\partial p}W^{-1}C\mathscr{A}\right\} + \mathscr{A}^\dagger\frac{\partial C}{\partial p}\mathscr{A}. \tag{16}$$

Further, assuming that the parameter p is contained in the passive part of the circuit, we have

$$\frac{\partial N_r}{\partial p} = 2\,\mathrm{Re}\left\{\mathscr{A}^\dagger\frac{\partial S}{\partial p}W^{-1}C\mathscr{A}\right\} - 2\mathscr{A}^\dagger\ \mathrm{Re}\left\{\frac{\partial S}{\partial p}S^\dagger\right\}\mathscr{A}. \tag{17}$$

Using this equation, we may write

$$G = \begin{bmatrix}\frac{\partial N_r}{\partial p_1}\\ \frac{\partial N_r}{\partial p_2}\\ \vdots \\ \frac{\partial N_r}{\partial p_n}\end{bmatrix} = 2\,\mathrm{Re}\left\{\mathscr{A}^\dagger\begin{bmatrix}\frac{\partial S}{\partial p_1}\\ \frac{\partial S}{\partial p_2}\\ \vdots \\ \frac{\partial S}{\partial p_n}\end{bmatrix}W^{-1}C\mathscr{A}\right\} - 2\mathscr{A}^\dagger\ \mathrm{Re}\left\{\begin{bmatrix}\frac{\partial S}{\partial p_1}\\ \frac{\partial S}{\partial p_2}\\ \vdots \\ \frac{\partial S}{\partial p_n}\end{bmatrix}S^\dagger\right\}\mathscr{A} \tag{18}$$

where $G = \nabla N_r$ is the vector of sensitivities of the noise power dissipated in the load at the rth port of the analyzed circuit.

Equations (17) and (18) relate changes in the noise power dissipated in the load to changes in the parameter values in the passive elements of the circuit.

In order to evaluate the noise power sensitivities, we need to know the partial derivatives of the element scattering matrices, the vector $\mathscr{A}$ given by the solution of the system of equations

$$W^\dagger\mathscr{A} = \mathscr{B}_r \tag{19}$$

and the inverse of the connection scattering matrix of the analyzed circuit.

The first two quantities are also used in the sensitivity analysis of circuits excited by sinusoidal signals. The sensitivity of sinusoidal wave a_r at the rth port is obtained from [13]–[15]:

$$\frac{\partial a_r}{\partial p} = \boldsymbol{\alpha}^T\frac{\partial S}{\partial p}\boldsymbol{a} \tag{20}$$

where $\boldsymbol{a}$ is a solution vector of the system of equations

$$W\boldsymbol{a} = \boldsymbol{c} \tag{21}$$

of the original circuit, and $\boldsymbol{\alpha}$ is the solution vector of the system of equations

$$W^T\boldsymbol{\alpha} = \boldsymbol{\gamma}_r \tag{22}$$

of the adjoined circuit, in which $\boldsymbol{\gamma}_r$ is the vector of the same form as the vector $\mathscr{B}_r$ given by (11).

Comparing (19) and (22), we find that the solution vectors of both systems of equations satisfy a relation

$$\mathscr{A} = \boldsymbol{\alpha}^* \tag{23}$$

which means that only one of the vectors needs to be found. This significantly reduces the computational effort when performing noise analysis, noise power sensitivity analysis, and sensitivity analysis for sinusoidal signals at the same time.

The partial derivatives of the element scattering matrices required for evaluation of noise power sensitivities can usually be found analytically. Table 12.1 in [13] presents scattering matrix partial derivatives for common design components with respect to useful parameters.

The inverse matrices W^{-1} and $(W^{-1})^\dagger$ of the connection scattering matrix W can be computed very effectively using sparse matrix technique based on the bifactorization method [16], [17].

IV. Noise Figure Gradient Computation

The noise figure of a circuit is given by

$$F = 1 + \frac{P_{N\,\mathrm{int}}}{P_{N_S}} \tag{24}$$

where $P_{N\,\mathrm{int}}$ is the (available) noise power at the output port of the circuit arising from the noise sources acting within the circuit, and P_{N_S} is the (available) noise power at the output port of the circuit arising from the equivalent thermal ($T_0 = 290$ K) noise source of the input port termination.

In the above definition of F it is assumed that the output port load impedance is noise free.

Because noise power at the output port of the analyzed circuit is given by (9) and (10), we have

$$F = 1 + \frac{N_{\mathrm{int}\,l}}{N_{s\,l}} \tag{25}$$

where, according to (13),

$$N_{\text{int}\,r} = \mathscr{A}^{\dagger} C_{\text{int}} \mathscr{A} \tag{26}$$

and

$$N_{sr} = \mathscr{A}^{\dagger} C_s \mathscr{A}. \tag{27}$$

The noise correlation matrices C_{int} and C_s are given as [1]

$$C_{\text{int}} = \begin{array}{c} \\ \\ \\ \\ p \\ \\ r \\ \\ \end{array}\begin{bmatrix} C^{(1)} & 0 & \cdot & & \vdots^{p} & \cdot & \vdots^{r} & \cdot & 0 \\ 0 & C^{(2)} & \cdot & & \vdots & & \vdots & & \cdot \\ \cdot & & & \cdot & \vdots & & \vdots & & \cdot \\ \cdots & \cdots & \cdot & \cdots & 0 & & \vdots & & \cdot \\ \cdot & & & & & \cdot\; C^{(k)} & \vdots & & \cdot \\ \cdots & \cdots & \cdot & \cdots & \cdot & \cdot & \cdots\; 0 & & \cdot \\ 0 & \cdot & & \cdot & \cdot & \cdot & & \cdot & C^{(m)} \end{bmatrix} \tag{28}$$

and

$$C_s = \begin{array}{c} \\ \\ \\ p \\ \\ \\ \\ \end{array}\begin{bmatrix} 0 & 0 & \cdot & \vdots^{p} & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & & \vdots & & & & \cdot \\ \cdot & & \cdot & \cdot & & & & \cdot \\ \cdots & \cdots & \cdots & C_{pp} & & & & \cdot \\ \cdot & & & & 0 & & & \cdot \\ \cdot & & & & & 0 & & \cdot \\ \cdot & & & & & & & \\ 0 & \cdot & & \cdot & \cdot & \cdot & \cdot & 0 \end{bmatrix} \tag{29}$$

In (28) two diagonal elements representing noise powers generated by the signal source impedance (port number p) and by the load impedance (port number r) are set to zero.

In (29) all elements are set to zero, except the element corresponding to the input port termination (signal source port).

Differentiating the relation (25) with respect to the circuit parameter p, using relations (26)–(29) and the relation (16) describing the noise power sensitivities, we get the expression for the noise figure sensitivity with respect to the parameter p:

$$\begin{aligned} \frac{\partial F}{\partial p} &= \frac{1}{N_{sr}^2}\left(\frac{\partial N_{\text{int}\,r}}{\partial p} N_{sr} - \frac{\partial N_{sr}}{\partial p} N_{\text{int}\,r}\right) \\ &= \frac{1}{\mathscr{A}^{\dagger} C_s \mathscr{A}}\left[2\,\text{Re}\left\{\mathscr{A}^{\dagger}\frac{\partial S}{\partial p} W^{-1} C_{\text{int}} \mathscr{A}\right\} + \mathscr{A}^{\dagger}\frac{\partial C_{\text{int}}}{\partial p}\mathscr{A}\right] \\ &\quad - \frac{\mathscr{A}^{\dagger} C_{\text{int}} \mathscr{A}}{\left(\mathscr{A}^{\dagger} C_s \mathscr{A}\right)^2} 2\,\text{Re}\left\{\mathscr{A}^{\dagger}\frac{\partial S}{\partial p} W^{-1} C_s \mathscr{A}\right\}. \end{aligned} \tag{30}$$

In deriving (30) we assumed that the thermal noise generated by the internal impedance of the signal generator did not depend on the circuit parameter p.

Consider now the design of a circuit using a gradient optimization method [13]. Suppose it is desired to minimize the noise figure of the circuit. Expanding the relation (30), we have the gradient vector of the noise figure F considered as the objective function of the gradient optimization procedure

$$\begin{aligned} \nabla F = \begin{bmatrix} \frac{\partial F_r}{\partial p_1} \\ \frac{\partial F_r}{\partial p_2} \\ \vdots \\ \frac{\partial F_r}{\partial p_n} \end{bmatrix} &= \frac{1}{\mathscr{A}^{\dagger} C_s \mathscr{A}}\left[2\,\text{Re}\left\{\mathscr{A}^{\dagger}\begin{bmatrix} \frac{\partial S}{\partial p_1} \\ \frac{\partial S}{\partial p_2} \\ \vdots \\ \frac{\partial S}{\partial p_n} \end{bmatrix} W^{-1} C_{\text{int}} \mathscr{A}\right\} + \mathscr{A}^{\dagger}\begin{bmatrix} \frac{\partial C_{\text{int}}}{\partial p_1} \\ \frac{\partial C_{\text{int}}}{\partial p_2} \\ \vdots \\ \frac{\partial C_{\text{int}}}{\partial p_n} \end{bmatrix}\mathscr{A}\right] \\ &\quad - \frac{\mathscr{A}^{\dagger} C_{\text{int}} \mathscr{A}}{\left(\mathscr{A}^{\dagger} C_s \mathscr{A}\right)^2} 2\,\text{Re}\left\{\mathscr{A}^{\dagger}\begin{bmatrix} \frac{\partial S}{\partial p_1} \\ \frac{\partial S}{\partial p_2} \\ \vdots \\ \frac{\partial S}{\partial p_n} \end{bmatrix} W^{-1} C_s \mathscr{A}\right\}. \end{aligned} \tag{31}$$

Equation (31) is a general one. It is applicable to cases where variable parameters are contained in passive as well as in active elements of the circuit.

The above discussion on computing the noise figure gradient vector can be extended to obtain a matrix of second-order derivatives (Hessian) of the noise figure F for a two-port with arbitrary internal topology.

V. Conclusion

The paper has presented theoretical work which facilitates the evaluation of noise power sensitivities of networks with any topology in a straightforward and computationally efficient manner. The noise power sensitivities are applicable for accurate and efficient gradient computation of the noise figure of microwave circuits. The theory presented is directly applicable to the computer-aided design of low-noise microwave circuits using gradient optimization methods. It is also applicable to two-ports with any internal topology and may be used in general-purpose computer programs for microwave circuit design.

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Equal-Gain Loci and Stability of a Microwave GaAs MESFET Gate Mixer

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Abstract—**The performance of a microwave GaAs MESFET gate mixer is theoretically investigated to clarify the existence of a conditionally stable RF frequency range as well as an unconditionally stable frequency range in which maximum available conversion gain (MACG) can be defined. For the unconditionally stable range, the MACG, and load and source impedances are calculated as functions of RF frequency. For the conditionally stable frequency range, the stability circle and equal gain loci are shown for source RF and load IF impedances. The conditionally stable region of the GaAs MESFET mixer appears around f_T of the MESFET. Higher conversion gain is easily obtained by choosing a MESFET of which the f_T is close to the RF frequency.**

I. Introduction

Since Pucel *et al.* reported the possibility of a GaAs MESFET mixer, many papers have demonstrated the advantages such a mixer confers with regard to conversion gain, better noise performance, and lower intermodulation products [1]–[4]. These papers have established that the mixer performance is determined by circuit conditions at RF, IF, local oscillator (LO), and image frequencies at both drain and gate ports. Better noise performance is obtained by shorting LO output, IF input, image input, and image output circuits [2]. Higher conversion gain is obtained by providing a conjugate matching condition for input RF and output IF circuits [1]–[3].

TABLE I
Parameter Values of the Nonlinear Elements

I_{ds}		I_{fg}	
a	0.26	I_s (pA)	4.72
b	0.75	α	28.4
m	2.96	linear element	
p	0.14	R_g (Ω)	1.4
V_p (V)	1.18	R_d (Ω)	1.6
V_{bi} (V)	0.90	R_s (Ω)	0.7
V_{dss} (V)	0.50	L_g (nH)	0.06
I_{dsp} (mA)	80.6	L_d (nH)	0.05
τ (ps)	4.74	L_s (nH)	0.6
C_{gs}		R_{ds} (Ω)	1093
ϕ (V)	0.11	C_{ds} (pF)	0.06
C_{gs0} (pF)	0.84	$\tau_i = R_i C_{gs}$ (ps)	1.2
F_c	−5.51	C_{gd} (pF)	0.05

However, there are still unknown factors. For example, no paper has yet described an RF frequency range in which instability may occur. This paper describes the calculated performance of a GaAs MESFET gate mixer to determine the conditionally stable RF frequency range. Separation of frequency ranges means that two different approaches are required in the design of the GaAs MESFET mixer. Thus, maximum available conversion gain (MACG), and load and source impedance will be calculated as functions of RF frequency in the unconditionally stable frequency range. In the conditionally stable range, equal gain loci will be calculated for RF source and IF load impedance.

II. Calculation Method of GaAs MESFET Gate Mixer Performance

Since the LO level is much higher than the RF level, the RF signal can be regarded as a perturbation superposed on the LO signal. Thus, large-signal drain and gate currents are calculated for the LO signal. We have calculated derivatives of periodically varying drain and gate currents with respect to gate and drain voltage. The conversion matrix [1] is determined using products of these derivatives and RF signal. In these calculations, the equivalent circuit for the GaAs MESFET is the same as that given in [5]. Three nonlinear elements are assumed: the drain current I_m, the gate–source capacitance C_{gs}, and the forward gate current I_{fg}. These elements are given by [5, eqs. (8)–(10)]. Parameter values of the nonlinear elements are listed in Table I and are determined by the following method. First, I_m, C_{gs}, and I_{fg} are fitted from dc $I-V$, $C_{gs}-V_{gs}$, and diode characteristics, respectively. Next, g_m, $\partial I_{ds}/\partial V_{ds}$, and C_{gs} are calculated at a bias using the nonlinear parameters. Calculated g_m and C_{gs} are compared with those determined from S parameters measured at the same bias. C_{gs} as determined by the former method is multiplied by a constant to set its value equal to that determined by the latter method. A correction to g_m is also made by multiplying another constant by the drain current determined by dc $I-V$. A linear parallel resistance R_{ds} is added to set the derivative of the drain current with respect to V_{ds} equal to that determined by the latter method. Parameters in Table I show corrected values. Remaining circuit elements are also listed in the table.

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